On contractions of classical basic superalgebras

2003 J. Phys. A: Math. Gen. 36 2483

(http://iopscience.iop.org/0305-4470/36/10/308)

INSTITUTE OF PHYSICS PUBLISHING JOURNAL OF PHYSICS A: MATHEMATICAL AND GENERAL

J. Phys. A: Math. Gen. **36** (2003) 2483–2492 PII: S0305-4470(03)55511-1

On contractions of classical basic superalgebras

N A Gromov, I V Kostyakov and V V Kuratov

Department of Mathematics, Syktyvkar Branch of IMM UrD RAS, Chernova st, 3a, Syktyvkar, 167982, Russia

E-mail: gromov@dm.komisc.ru

Received 28 October 2002, in final form 23 January 2003
Published 26 February 2003
Online at stacks.iop.org/JPhysA/36/2483

Abstract

We define a class of orthosymplectic $osp(m; j|2n; \omega)$ and unitary $sl(m; j|n; \epsilon)$ superalgebras which may be obtained from $osp(m|2n)$ and $sl(m|n)$ by contractions and analytic continuations in a similar way as the special linear, orthogonal and the symplectic Cayley–Klein algebras are obtained from the corresponding classical ones. Casimir operators of Cayley–Klein superalgebras are obtained from the corresponding operators of the basic superalgebras. Contractions of $sl(2|1)$ and $osp(3|2)$ are regarded as examples.

PACS numbers: 02.20.−a, 02.10.Hh, 11.30.Pb

1. Introduction

Since its discovery [1–3] in 1971 supersymmetry has been used in different physical theories such as Kaluza–Klein supergravity [4], supersymmetric field theories of the Wess–Zumino type [5, 6] and massless higher-spin field theories [7]. Recently the secret theory [8] (or S-theory) that includes superstring theory and its super p-brane and D-brane [9] generalizations were discussed. All these theories are built algebraically with the help of some superalgebra as their base. In this work we wish to present a wide class of Cayley–Klein (CK) superalgebras which may be used for the construction of different supersymmetric models.

For ordinary Lie groups (or algebras) the title CK was initially used as the short name for the set of motion groups of spaces of constant curvature. It is well known that there are 3^n n-dimensional spaces of constant curvature and their motion groups may be obtained from the orthogonal group $SO(n+1)$ with the help of contractions and analytical continuations [10]. Later the notion CK was extended to the case of unitary and symplectic groups (algebras) [11]. The typical (and attractive) property of CK groups is that all of them depend on the same number of independent parameters as the corresponding simple classical group. At the level of Lie algebras this means that all CK algebras of the same type have the same dimensions. Basic superalgebras include simple algebras as even subalgebras, so it looks quite natural to introduce a new class of superalgebras with CK algebras as even subalgebras.

A superalgebra as an algebraic structure contains (as compared with Lie algebra) a new additional operation, namely, Z_2-grading. So under the contraction of superalgebra this Z_2-grading must be conserved. To our knowledge contraction of an orthosymplectic superalgebra to the superkinematics was first discussed in [12]. The detailed investigation of a class of contractions of $osp(1|2)$ and $osp(1|4)$ to the kinematical Poincaré and Galilei superalgebras was made in [13]. Contraction of unitary superalgebra $Gsu(2) = sl(2|1)$ as well as their representations was described in [14]. Later the notion of contraction was generalized [15, 16] to the case of Lie algebra and superalgebra with an arbitrary finite grading group and is known as graded contractions. A new kind of discrete contraction was defined and also found in these papers. Nevertheless the particular case of the simplest Z_2-grading deserves an independent interest. It should be mentioned that contractions of quantum deformations of superalgebras mentioned in [17] form a separate line of investigation. In connection with a superstring theory in anti-de Sitter space the generalized Inönü–Wigner contraction was proposed [6], which keeps the next-to-leading term of a contraction parameter and gives a correct flat limit of the Wess–Zumino term. We deal with the standard Inönü–Wigner contractions [18], but we use nilpotent-valued contraction parameters instead of zero-tending ones. Our preliminary results were reported in [19].

The paper is organized as follows. In section 2, the orthogonal, symplectic and special linear CK groups and algebras are briefly recalled. Section 3 is devoted to the orthosymplectic CK superalgebras. CK unitary superalgebras are discussed in section 4. Casimir operators of the CK unitary and orthosymplectic superalgebras are described in section 5.

2. Orthogonal, symplectic and special linear Cayley–Klein algebras

Special linear $sl(m)$, orthogonal $so(m)$ and symplectic $sp(2n)$ algebras are even subalgebras of classical basic superalgebras. On the other hand, all of them may be contracted and analytically continued to the set of CK algebras. Lie groups and algebras are in close relations. The CK group $SO(m; j)$ is defined as the set of transformations of vector space $\mathbf{R}_m(j)$, which preserve the quadratic form $x^2(j) = x^t(j)x(j) = x_1^2 + \sum_{k=2}^{m}(1,k)^2x_k^2$, where $(i,k) = \prod_{p=\min(i,k)}^{\max(i,k)-1} j_p$, $(i,i) = 1$, each parameter $j_k = 1, \iota_k, i$, where ι_k are nilpotent $\iota_k^2 = 0$, commutative $\iota_k\iota_p = \iota_p\iota_k \neq 0$ generators of Pimenov algebra $P(\iota)$. The Cartesian components of vector $x(j) \in \mathbf{R}_m(j)$ are $x^t(j) = (x_1, j_1x_2, \ldots, (1,m)x_m)^t$, as easily follows from $x^2(j)$. For an $m \times m$ matrix $g(j) \in SO(m; j)$ the transformation $g(j) : \mathbf{R}_m(j) \to \mathbf{R}_m(j)$ means that the vector $x'(j) = g(j)x(j)$ has exactly the same distribution of parameters j among its components as $x(j)$. This requirement gives an opportunity to obtain the distribution of parameters j among the elements of matrix $g(j)$, i.e. to build the fundamental representation of the CK group $SO(m; j)$ starting from the quadratic form. It is remarkable that the same distribution of parameters j holds for the CK Lie algebra $so(m; j)$, namely $A_{ik} = (i,k)a_{ik}$, for $A \in so(m; j)$.

For the simple case $m = 3$ the nine CK plane geometries (or geometries of constant curvature) are realized on the spheres $S_2(j) = \left\{x_1^2 + j_1^2x_2^2 + j_1^2j_2^2x_3^2 = 1\right\}$ in the spaces $R_3(j)$ with Cartesian coordinates x_1, j_1x_2, $j_1j_2x_3$. The identifications of the well-known plane names and the values of parameters j_1, j_2 are given in table 1. The motion group of the geometry $S_2(j)$ is isomorphic with the following matrix group $SO(3; j) = \{g(j) | \det g(j) = 1, g^t(j)g(j) = I\}$, whose elements are nilpotent under nilpotent values of j. It is worth remarking that the same motion group can also be described with the help of the *real* matrix group $SO(3; j) = \{\tilde{g}(j) | \det \tilde{g}(j) = 1, \tilde{g}^t(j)\tilde{g}(j) = I\}$, where matrices $g(j)$ and $\tilde{g}(j)$ are as follows:

Table 1. The nine Cayley–Klein plane geometries.

	$j_1 = 1$	$j_1 = \iota_1$	$j_1 = i$
$j_2 = 1$	Elliptic	Euclidean	Lobachevskian (hyperbolic)
$j_2 = \iota_2$	Semielliptic (co-Euclidean)	Galilean	Semihyperbolic (co-Minkowskian)
$j_2 = i$	Anti-de Sitter	Minkowskian	de Sitter

$$g(j) = \begin{pmatrix} g_{11} & j_1 g_{12} & j_1 j_2 g_{13} \\ j_1 g_{21} & g_{22} & j_2 g_{23} \\ j_1 j_2 g_{31} & j_2 g_{32} & g_{33} \end{pmatrix} \qquad \tilde{g}(j) = \begin{pmatrix} \tilde{g}_{11} & j_1^2 \tilde{g}_{12} & j_1^2 j_2^2 \tilde{g}_{13} \\ \tilde{g}_{21} & \tilde{g}_{22} & j_2^2 \tilde{g}_{23} \\ \tilde{g}_{31} & \tilde{g}_{32} & \tilde{g}_{33} \end{pmatrix}.$$

The rotation matrix generators of the CK algebra $so(3; j)$ are given by

$$E_{12} = j_1 \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad E_{13} = j_1 j_2 \begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \qquad E_{23} = j_2 \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}$$

and are the subject of the commutation relations $[H, P] = j_1^2 K, [P, K] = j_2^2 H, [H, K] = -P$, where $H = E_{12}$ is the generator of time translation, $P = E_{13}$ is the generator of space translation and $K = E_{23}$ is the generator of Galilei $(j_2 = \iota_2)$ or Lorentz $(j_2 = i)$ boost in the cases when CK spaces $S_2(j)$ may be interpreted as kinematics. More details on CK geometries and their motion groups are, for example, given in [20].

The set of transformations $L(j) : \mathbf{R}_m(j) \to \mathbf{R}_m(j)$ with the property $\det L(j) = 1$ forms the CK special linear group $SL(m;\, j)$ and the corresponding CK algebras $sl(m;\, j)$ are given by the $m \times m$ matrices $l(j)$, $\operatorname{tr} l(j) = 0$. Let us stress that in the Cartesian basis all matrices from $SL(m;\, j), SO(m;\, j), sl(m;\, j), so(m;\, j)$ have identical distribution of parameters j among their elements, i.e. they are of the same type as the matrices with elements from Pimenov algebra $P(j)$.

The CK symplectic group $Sp(2n; \omega)$ is defined as the set of transformations of $\mathbf{R}_n(\omega) \times \mathbf{R}_n(\omega)$, which preserve the bilinear form $S(\omega) = S_1 + \sum_{k=2}^{n} [1, k]^2 S_k$, where $S_k(y, z) = y_k z_{n+k} - y_{n+k} z_k$, $[i, k] = \prod_{p=\min(i,k)}^{\max(i,k)-1} \omega_k$, $[i, i] = 1$, $\omega_k = 1, \xi_k, i, \xi_k^2 = 0, \xi_k \xi_p = \xi_p \xi_k$. The distribution of parameters ω_k among the matrix elements of the fundamental representation $M(\omega) = \begin{pmatrix} H(\omega) & E(\omega) \\ F(\omega) & -H^t(\omega) \end{pmatrix}$ of the CK symplectic algebra $sp(2n; \omega)$ may be obtained as for the orthogonal CK algebras and is as follows: $B_{ik} = [i, k] b_{ik}$, where $B = H(\omega), E(\omega), F(\omega)$.

3. Orthosymplectic superalgebras $osp(m; j|2n; \omega)$

Let $e_{IJ} \in M_{m+2n}$ satisfying $(e_{IJ})_{KL} = \delta_{IK}\delta_{JL}$ be elementary matrices. One defines the following graded matrix [21]:

$$G = \left(\begin{array}{c|cc} I_m & \multicolumn{2}{c}{0} \\ \hline 0 & 0 & I_n \\ & -I_n & 0 \end{array}\right) \tag{1}$$

where I_m, I_n are identity matrices. Let $i, j, \ldots = 1, \ldots, m, \bar{i}, \bar{j}, \ldots = m+1, \ldots, m+2n$. The generators of the orthosymplectic superalgebra $osp(m|2n)$ are given by

$$
\begin{aligned}
E_{ij} &= -E_{ji} = \sum_k (G_{ik}e_{kj} - G_{jk}e_{ki}) \\
E_{\bar{i}\bar{j}} &= E_{\bar{j}\bar{i}} = \sum_{\bar{k}} (G_{\bar{i}\bar{k}}e_{\bar{k}\bar{j}} + G_{\bar{j}\bar{k}}e_{\bar{k}\bar{i}}) \\
E_{i\bar{j}} &= E_{\bar{j}i} = \sum_k G_{ik}e_{k\bar{j}} + \sum_{\bar{k}} G_{\bar{j}\bar{k}}e_{\bar{k}i}
\end{aligned} \tag{2}
$$

where the even (bosonic) E_{ij} generate the $so(m)$ part, the even (bosonic) $E_{\bar{i}\bar{j}}$ generate the $sp(2n)$ part and the rest, $E_{i\bar{j}}$, are the odd (fermionic) generators of superalgebra. They satisfy the following (super)commutation relations:

$$
\begin{aligned}
&[E_{ij}, E_{kl}] = G_{jk}E_{il} + G_{il}E_{jk} - G_{ik}E_{jl} - G_{jl}E_{ik} \\
&[E_{\bar{i}\bar{j}}, E_{\bar{k}\bar{l}}] = -G_{\bar{j}\bar{k}}E_{\bar{i}\bar{l}} - G_{\bar{i}\bar{l}}E_{\bar{j}\bar{k}} - G_{\bar{j}\bar{l}}E_{\bar{i}\bar{k}} - G_{\bar{i}\bar{k}}E_{\bar{j}\bar{l}} \\
&[E_{ij}, E_{k\bar{l}}] = G_{jk}E_{i\bar{l}} - G_{ik}E_{j\bar{l}}, \; [E_{i\bar{j}}, E_{\bar{k}\bar{l}}] = -G_{\bar{j}\bar{k}}E_{i\bar{l}} - G_{\bar{j}\bar{l}}E_{i\bar{k}} \\
&[E_{ij}, E_{\bar{k}\bar{l}}] = 0 \qquad \{E_{i\bar{j}}, E_{k\bar{l}}\} = G_{ik}E_{\bar{j}\bar{l}} - G_{\bar{j}\bar{l}}E_{ik}.
\end{aligned} \tag{3}
$$

In the matrix form $osp(m|2n) = \{M \in M_{m+2n} | M^{st}G + GM = 0\}$. If the matrix M has the following form: $M = \sum_{i,j} a_{ij}E_{ij} + \sum_{\bar{i},\bar{j}} b_{\bar{i}\bar{j}}E_{\bar{i}\bar{j}} + \sum_{i\bar{j}} \mu_{i\bar{j}}E_{i\bar{j}}$, with $a_{ij}, b_{\bar{i}\bar{j}} \in \mathbf{R}$ or $\mathbf{C}$ and $\mu_{i\bar{j}}$ as the odd nilpotent elements of the Grassmann algebra: $\mu^2_{i\bar{j}} = 0$, $\mu_{i\bar{j}}\mu_{i'\bar{j}'} = -\mu_{i'\bar{j}'}\mu_{i\bar{j}}$, then the corresponding supergroup $OSp(m|2n)$ is obtained by the exponential map $\mathcal{M} = \exp M$ and acts on (super)vector space by matrix multiplication $\mathcal{X}' = \mathcal{M}\mathcal{X}$, where $\mathcal{X}^t = (x|\theta)^t$, x is an n-dimensional even vector and θ is a $2m$-dimensional odd vector with odd Grassmann elements. The form $\mathrm{inv} = \sum_{i=1}^m x_i^2 + 2\sum_{k=1}^n \theta_{+k}\theta_{-k} = x^2 + 2\theta^2$ is invariant under this action of the orthosymplectic supergroup.

We shall define CK orthosymplectic superalgebras starting with the invariant form

$$
\mathrm{inv}(j;\omega) = u^2 \sum_{k=1}^m (1,k)^2 x_k^2 + 2v^2 \sum_{\bar{k}=m+1}^{m+n} [1,\hat{\bar{k}}]^2 \theta_{\hat{\bar{k}}}\theta_{-\hat{\bar{k}}} \equiv u^2x^2(j) + 2v^2\theta^2(\omega) \tag{4}
$$

$\hat{\bar{k}} = \bar{k} - m$, when $\bar{k} = m+1, \ldots, m+n$ and $\hat{\bar{k}} = \bar{k} - m - n$, when $\bar{k} = m+n+1, \ldots, m+2n$, which is the natural unification of the CK orthogonal and symplectic forms. The distributions of contraction parameters j, ω among the matrix elements of the fundamental representation of $osp(m; j|2n; \omega)$ and the transformations of the generators (2) are obtained in a standard CK manner and are as follows:

$$
E_{ik} = (i,k)E^\star_{ik} \qquad E_{\bar{i}\bar{k}} = [\hat{\bar{i}}, \hat{\bar{k}}]E^\star_{\bar{i}\bar{k}} \qquad E_{i\bar{k}} = u(1,i)v[1,\hat{\bar{k}}]E^\star_{i\bar{k}} \tag{5}
$$

where $E^\star$ are generators (2) of the starting superalgebra $osp(m|2n)$. The transformed generators are the subject of the (super)commutation relations:

$$
\begin{aligned}
&[E_{ij}, E_{kl}] = (i,j)(k,l)\left(\frac{G_{jk}E_{il}}{(i,l)} + \frac{G_{il}E_{jk}}{(j,k)} - \frac{G_{ik}E_{jl}}{(j,l)} - \frac{G_{jl}E_{ik}}{(i,k)}\right) \\
&[E_{\bar{i}\bar{j}}, E_{\bar{k}\bar{l}}] = -[\hat{\bar{i}}, \hat{\bar{j}}][\hat{\bar{k}}, \hat{\bar{l}}]\left(\frac{G_{\bar{j}\bar{k}}E_{\bar{i}\bar{l}}}{[\hat{\bar{i}},\hat{\bar{l}}]} + \frac{G_{\bar{i}\bar{l}}E_{\bar{j}\bar{k}}}{[\hat{\bar{j}},\hat{\bar{k}}]} + \frac{G_{\bar{i}\bar{k}}E_{\bar{j}\bar{l}}}{[\hat{\bar{j}},\hat{\bar{l}}]} + \frac{G_{\bar{j}\bar{l}}E_{\bar{i}\bar{k}}}{[\hat{\bar{i}},\hat{\bar{k}}]}\right) \\
&[E_{ij}, E_{\bar{k}\bar{l}}] = 0 \qquad [E_{ij}, E_{k\bar{l}}] = (i,j)(1,k)\left(\frac{G_{jk}E_{i\bar{l}}}{(1,i)} - \frac{G_{ik}E_{j\bar{l}}}{(1,j)}\right) \\
&[E_{i\bar{j}}, E_{\bar{k}\bar{l}}] = -[1,\hat{\bar{j}}][\hat{\bar{k}},\hat{\bar{l}}]\left(\frac{G_{\bar{j}\bar{k}}E_{i\bar{l}}}{[1,\hat{\bar{l}}]} + \frac{G_{\bar{j}\bar{l}}E_{i\bar{k}}}{[1,\hat{\bar{k}}]}\right) \\
&\{E_{i\bar{j}}, E_{k\bar{l}}\} = u^2v^2(1,i)[1,\hat{\bar{j}}](1,k)[1,\hat{\bar{l}}]\left(\frac{G_{ik}E_{\bar{j}\bar{l}}}{[\hat{\bar{j}},\hat{\bar{l}}]} - \frac{G_{\bar{j}\bar{l}}E_{ik}}{(i,k)}\right).
\end{aligned} \tag{6}
$$

For the $u = \iota$ or $v = \iota, \iota^2 = 0$ superalgebra $osp(m|2n)$ is contracted to an inhomogeneous superalgebra, which is the semidirect sum $\{E_{i\bar{j}}\} \ni (so(m) \oplus sp(2n))$, with all anticommutators of the odd generators equal to zero $\{E_{i\bar{j}}, E_{k\bar{p}}\} = 0$.

3.1. Example: CK contractions of $osp(3|2)$

This superalgebra has $so(3)$ as even subalgebra therefore the contractions to the kinematical (1 + 1) Poincaré, Newton and Galilei superalgebras may be fulfilled according to the general CK scheme of section 1. But unlike the two odd generators of $osp(1|2)$, the superalgebra $osp(3|2)$ has six odd generators. In the basis $X_{ik} = E_{ki}, k, i = 1, 2, 3, F = \frac{1}{2}E_{44}, E = -\frac{1}{2}E_{55}, H = -E_{45}, Q_k = E_{k4}, Q_{-k} = E_{k5}$ the generators are affected by the contraction coefficients j_1, j_2 as follows:

$$X_{ik} \to (i,k)X_{ik} \qquad Q_{\pm k} \to (1,k)Q_{\pm k} \tag{7}$$

and H, F, E remain unchanged. Then superalgebra $osp(3; j|2)$ is given by

$$\begin{array}{lll}
[X_{12}, X_{13}] = j_1^2 X_{23} & [X_{13}, X_{23}] = j_2^2 X_{12} & [X_{23}, X_{12}] = X_{13} \\
[H, E] = 2E & [H, F] = -2F & [E, F] = H \\
[X_{ik}, Q_{\pm i}] = Q_{\pm k} & [X_{ik}, Q_{\pm k}] = -(i,k)^2 Q^2_{\pm i} & i < k \\
[H, Q_{\pm k}] = \mp Q_{\pm k} & [E, Q_k] = -Q_{-k} & [F, Q_{-k}] = -Q_k \\
\{Q_k, Q_k\} = (1,k)^2 F & \{Q_{-k}, Q_{-k}\} = -(1,k)^2 E & \\
\{Q_k, Q_{-k}\} = -(1,k)^2 H & \{Q_{\pm i}, Q_{\mp k}\} = \pm(1,k)^2 X_{ik}. &
\end{array} \tag{8}$$

The non-minimal Poincaré superalgebra is obtained for $j_1 = \iota_1$, $j_2 = i$ and has the structure of the semidirect sum $T \ni (\{X_{23}\} \oplus osp(1|2))$, where Abelian $T = \{X_{12}, X_{13}, Q_{\pm 2}, Q_{\pm 3}\}$ and $osp(1|2) = \{H, E, F, Q_{\pm 1}\}$. The Newton superalgebra $osp(3; \iota_2|2) = T_2 \ni osp(2|2)$, where $T_2 = \{X_{13}, X_{23}, Q_{\pm 3}\}$ and $osp(2|2)$ is generated by $X_{12}, H, E, F, Q_{\pm 1}, Q_{\pm 2}$. Finally the non-minimal Galilei superalgebra may be presented as semidirect sums $osp(3; \iota_1, \iota_2|2) = (T \ni \{X_{23}\}) \ni osp(1|2) = T \ni (\{X_{23}\} \oplus osp(1|2))$.

4. Unitary superalgebras $sl(m; j|n; \epsilon)$

The superalgebras $sl(m|n)$ can be generated as matrix superalgebras by taking matrices of the form [21]

$$M = \begin{pmatrix} X_{mm} & T_{mn} \\ T_{nm} & X_{nn} \end{pmatrix} \tag{9}$$

where X_{mm} and X_{nn} are $gl(m)$ and $gl(n)$ matrices, T_{mn} and T_{nm} are $m \times n$ and $n \times m$ matrices, respectively, with the supertrace condition

$$\mathrm{str}(M) = \mathrm{tr}(X_{mm}) - \mathrm{tr}(X_{nn}) = 0. \tag{10}$$

This matrix superalgebra is the set of transformations of the superspace with m even coordinates $x_1, \ldots, x_m$ and n odd ones $\theta_1, \ldots, \theta_n$.

A basis of superalgebra $sl(m|n)$ can be constructed as follows. Define the $(m + n)^2 - 1$ generators

$$\begin{array}{ll}
E_{ij} = e_{ij} - \dfrac{1}{m-n}\delta_{ij}\left(\displaystyle\sum_{k=1}^{m} e_{kk} + \sum_{\bar{k}=m+1}^{m+n} e_{\bar{k}\bar{k}}\right) & E_{i\bar{j}} = e_{i\bar{j}} \\
E_{\bar{i}\bar{j}} = e_{\bar{i}\bar{j}} + \dfrac{1}{m-n}\delta_{\bar{i}\bar{j}}\left(\displaystyle\sum_{k=1}^{m} e_{kk} + \sum_{\bar{k}=m+1}^{m+n} e_{\bar{k}\bar{k}}\right) & E_{\bar{i}j} = e_{\bar{i}j}
\end{array} \tag{11}$$

where the indices $i, j, \ldots$ run from 1 to m and $\bar{i}, \bar{j}, \ldots$ from $m+1$ to $m+n$. The generators of $sl(m|n)$ in the Cartan–Weyl basis are given by

$$\begin{aligned}
&H_i = E_{ii} - E_{i+1,i+1} \qquad 1 \leqslant i \leqslant m-1 \\
&H_{\bar{i}} = E_{\bar{i}\bar{i}} - E_{\bar{i}+1,\bar{i}+1} \qquad m+1 \leqslant \bar{i} \leqslant m+n-1 \\
&H_m = E_{mm} + E_{m+1,m+1} \\
&E_{ij} \quad \text{for} \quad sl(m) \qquad E_{\bar{i}\bar{j}} \quad \text{for} \quad sl(n) \\
&E_{i\bar{j}} \quad \text{and} \quad E_{\bar{i}j} \quad \text{for the odd part}
\end{aligned} \tag{12}$$

and their commutation relations appear as

$$\begin{aligned}
&[H_I, H_J] = 0 \\
&[H_K, E_{IJ}] = \delta_{IK}E_{KJ} - \delta_{I,K+1}E_{K+1,J} - \delta_{KJ}E_{IK} + \delta_{K+1,J}E_{I,K+1} \qquad (K \neq m) \\
&[H_m, E_{IJ}] = \delta_{Im}E_{mJ} - \delta_{I,m+1}E_{m+1,J} - \delta_{mJ}E_{Im} + \delta_{m+1,J}E_{I,m+1} \\
&[E_{IJ}, E_{KL}] = \delta_{JK}E_{IL} - \delta_{IL}E_{KJ} \qquad \text{for} \quad E_{IJ} \text{ and } E_{KL} \text{ even} \\
&[E_{IJ}, E_{KL}] = \delta_{JK}E_{IL} - \delta_{IL}E_{KJ} \qquad \text{for} \quad E_{IJ} \text{ even and } E_{KL} \text{ odd} \\
&\{E_{IJ}, E_{KL}\} = \delta_{JK}E_{IL} + \delta_{IL}E_{KJ} \qquad \text{for} \quad E_{IJ} \text{ and } E_{KL} \text{ odd.}
\end{aligned} \tag{13}$$

CK special linear (or unitary) superalgebras $sl(m; j|n; \epsilon)$ are consistent with the transformations of (super)vectors

$$\mathcal{X}^t(j,\epsilon) = (x_1, j_1x_2, \ldots, (1,m)x_m|\nu(x_{m+1}, \epsilon_1x_{m+2}, \ldots, [1,n]x_{m+n}))^t \tag{14}$$

where the odd components are denoted as $x_{m+1} = \theta_1, \ldots, x_{m+n} = \theta_n$ and $\hat{\hat{i}} = \bar{i} - m, \hat{\hat{k}} = \bar{k} - m = 1, \ldots, n, [\hat{\hat{i}}, \hat{\hat{k}}] = \prod_{l=\min(\hat{\hat{i}},\hat{\hat{k}})}^{\max(\hat{\hat{i}},\hat{\hat{k}})-1} \epsilon_l, \epsilon_l = 1, \xi_l, i, \xi_l^2 = 0, \xi_l\xi_p = \xi_p\xi_l \neq 0$. The components of $\mathcal{X}(j;\epsilon)$ are chosen in such a way that the contraction parameters ϵ_l of the odd components are independent of the contraction parameters j_l of the even ones. The transformations of the standard generators (12) (marked with a star) of the special linear superalgebra $sl(m|n)$ to the generators of $sl(m; j|n, \epsilon)$ are given by

$$\begin{aligned}
&H_I = H_I^\star \qquad E_{ij} = (i,j)E_{ij}^\star \qquad E_{\bar{i}\bar{j}} = [\hat{\hat{i}}, \hat{\hat{j}}]E_{\bar{i}\bar{j}}^\star \qquad i \neq j \quad \bar{i} \neq \bar{j} \\
&E_{i\bar{j}} = \nu(1,i)[1,\hat{\hat{j}}]E_{i\bar{j}}^\star \qquad E_{\bar{i}j} = \nu(1,j)[1,\hat{\hat{i}}]E_{\bar{i}j}^\star.
\end{aligned} \tag{15}$$

Nonzero commutators and anticommutators are easily obtained from the corresponding commutation relations (13) of the initial superalgebra $sl(m|n)$ in the form

$$[H_K, E_{IJ}] = \delta_{IK}E_{KJ} - \delta_{I,K+1}E_{K+1,J} - \delta_{KJ}E_{IK} + \delta_{K+1,J}E_{I,K+1}$$

$$[E_{ij}, E_{jl}] = \begin{cases} E_{il} & i < j < l \quad l < j < i \quad l \neq i \\ (l,j)^2E_{il} & i < l < j \quad \text{or} \quad j < l < i \\ (i,j)^2E_{il} & l < i < j \quad \text{or} \quad j < i < l \end{cases}$$

$$[E_{ij}, E_{kj}] = \begin{cases} -E_{kj} & k < i < j \quad j < i < k \quad k \neq j \\ -(i,j)^2E_{kj} & i < j < k \quad \text{or} \quad k < j < i \\ -(i,k)^2E_{kj} & i < k < j \quad \text{or} \quad j < k < i \end{cases}$$

$$[E_{ij}, E_{ji}] = (i,j)^2(E_{ii} - E_{jj})$$

$$[E_{\bar{i}\bar{j}}, E_{\bar{j}\bar{l}}] = \begin{cases} E_{\bar{i}\bar{l}} & \bar{i} < \bar{j} < \bar{l} \quad \bar{l} < \bar{j} < \bar{i} \quad \bar{l} \neq \bar{i} \\ [\hat{\hat{l}}, \hat{\hat{j}}]^2E_{\bar{i}\bar{l}} & \bar{i} < \bar{l} < \bar{j} \quad \text{or} \quad \bar{j} < \bar{l} < \bar{i} \\ [\hat{\hat{i}}, \hat{\hat{j}}]^2E_{\bar{i}\bar{l}} & \bar{l} < \bar{i} < \bar{j} \quad \text{or} \quad \bar{j} < \bar{i} < \bar{l} \end{cases}$$

$$[E_{\bar{i}\bar{j}}, E_{\bar{k}\bar{j}}] = \begin{cases} -E_{\bar{k}\bar{j}} & \bar{k} < \bar{i} < \bar{j} \quad \bar{j} < \bar{i} < \bar{k} \quad \bar{k} \neq \bar{j} \\ -[\hat{\bar{i}}, \hat{\bar{j}}]^2 E_{\bar{k}\bar{j}} & \bar{i} < \bar{j} < \bar{k} \quad \text{or} \quad \bar{k} < \bar{j} < \bar{i} \\ -[\hat{\bar{i}}, \hat{\bar{k}}]^2 E_{\bar{k}\bar{j}} & \bar{i} < \bar{k} < \bar{j} \quad \text{or} \quad \bar{j} < \bar{k} < \bar{i} \end{cases}$$

$$[E_{\bar{i}\bar{j}}, E_{\bar{j}\bar{i}}] = [\hat{\bar{i}}, \hat{\bar{j}}]^2 (E_{\bar{i}\bar{i}} - E_{\bar{j}\bar{j}})$$

$$[E_{ij}, E_{j\bar{l}}] = \begin{cases} (i, j)^2 E_{i\bar{l}} & i < j \\ E_{i\bar{l}} & i > j \end{cases} \qquad [E_{ij}, E_{\bar{k}i}] = \begin{cases} -E_{\bar{k}j} & i < j \\ -(j, i)^2 E_{\bar{k}j} & i > j \end{cases}$$

$$[E_{\bar{i}\bar{j}}, E_{k\bar{i}}] = \begin{cases} -E_{k\bar{j}} & \bar{i} < \bar{j} \\ -[\hat{\bar{j}}, \hat{\bar{i}}]^2 E_{k\bar{j}} & \bar{i} > \bar{j} \end{cases} \qquad [E_{\bar{i}\bar{j}}, E_{\bar{j}l}] = \begin{cases} [\hat{\bar{i}}, \hat{\bar{j}}]^2 E_{\bar{i}l} & \bar{i} < \bar{j} \\ E_{\bar{i}l} & \bar{i} > \bar{j} \end{cases}$$

$$\{E_{i\bar{j}}, E_{\bar{j}l}\} = \begin{cases} \nu^2 [1, \hat{\bar{j}}]^2 (1, i)^2 E_{il} & i < l \\ \nu^2 [1, \hat{\bar{j}}]^2 (1, l)^2 E_{il} & i > l \end{cases}$$

$$\{E_{i\bar{j}}, E_{\bar{k}i}\} = \begin{cases} \nu^2 (1, i)^2 [1, \hat{\bar{j}}]^2 E_{\bar{k}\bar{j}} & \bar{j} < \bar{k} \\ \nu^2 (1, i)^2 [1, \hat{\bar{k}}]^2 E_{\bar{k}\bar{j}} & \bar{j} > \bar{k} \end{cases}$$

$$\{E_{i\bar{j}}, E_{\bar{j}i}\} = \nu^2 (1, i)^2 [1, \hat{\bar{j}}]^2 (E_{ii} + E_{\bar{j}\bar{j}}). \tag{16}$$

For $\nu = \iota$ superalgebra $sl(m|n)$ is contracted to an inhomogeneous superalgebra, which is the semidirect sum $\{E_{i\bar{j}}, E_{\bar{i}j}\} \ni (sl(m) \oplus sl(n))$, with all anticommutators of the odd generators equal to zero.

4.1. Example: CK contractions of sl(2|1)

The generators of superalgebra $sl(2; j_1; \nu|1)$ are given by [21]

$$\begin{aligned} H &= \left(\begin{array}{cc|c} \frac{1}{2} & 0 & 0 \\ 0 & -\frac{1}{2} & 0 \\ \hline 0 & 0 & 0 \end{array}\right) & Z &= \left(\begin{array}{cc|c} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ \hline 0 & 0 & 1 \end{array}\right) \\ E_{12} = E^+ &= \left(\begin{array}{cc|c} 0 & j_1 & 0 \\ 0 & 0 & 0 \\ \hline 0 & 0 & 0 \end{array}\right) & E_{21} = E^- &= \left(\begin{array}{cc|c} 0 & 0 & 0 \\ j_1 & 0 & 0 \\ \hline 0 & 0 & 0 \end{array}\right) \\ E_{13} = \bar{F}^+ &= \left(\begin{array}{cc|c} 0 & 0 & \nu \\ 0 & 0 & 0 \\ \hline 0 & 0 & 0 \end{array}\right) & E_{31} = F^- &= \left(\begin{array}{cc|c} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \hline \nu & 0 & 0 \end{array}\right) \\ E_{32} = F^+ &= \left(\begin{array}{cc|c} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \hline 0 & \nu j_1 & 0 \end{array}\right) & E_{23} = \bar{F}^- &= \left(\begin{array}{cc|c} 0 & 0 & 0 \\ 0 & 0 & \nu j_1 \\ \hline 0 & 0 & 0 \end{array}\right) \end{aligned} \tag{17}$$

and act on the superspace $(x_1, j_1 x_2 | \nu\theta_1)$. The commutation relations are represented as

$$\begin{aligned} &[H, E^{\pm}] = \pm E^{\pm} \qquad [E^+, E^-] = 2j_1^2 H \\ &[Z, H] = [Z, E^{\pm}] = [E^{\pm}, \bar{F}^{\pm}] = [E^{\pm}, F^{\pm}] = 0 \\ &[H, \bar{F}^{\pm}] = \pm\tfrac{1}{2}\bar{F}^{\pm} \qquad [H, F^{\pm}] = \pm\tfrac{1}{2}F^{\pm} \qquad [Z, F^{\pm}] = \tfrac{1}{2}F^{\pm} \qquad [Z, \bar{F}^{\pm}] = -\tfrac{1}{2}\bar{F}^{\pm} \\ &[E^+, F^-] = -F^+ \qquad [E^-, F^+] = -j_1^2 F^- \qquad [E^+, \bar{F}^-] = j_1^2 \bar{F}^+ \qquad [E^-, \bar{F}^+] = \bar{F}^- \\ &\{F^+, \bar{F}^-\} = \nu^2 j_1^2 (Z - H) \qquad \{F^-, \bar{F}^+\} = \nu^2 (Z + H) \\ &\{\bar{F}^+, F^+\} = \nu^2 E^+ \qquad \{\bar{F}^-, F^-\} = \nu^2 E^- \qquad \{\bar{F}^+, \bar{F}^-\} = \{F^+, F^-\} = 0. \end{aligned} \tag{18}$$

For $\nu = \iota$ we obtain the semidirect sum of the Abelian odd subalgebra with the direct sum of the even subalgebras, namely, $sl(2; j_1; \iota|1) = \{F^{\pm}, \bar{F}^{\pm}\} \ni (u(1) \oplus sl(2))$. The two-dimensional contraction $\nu = \iota, j_1 = \iota_1$ results in similar semidirect sum $sl(2; \iota_1; \iota|1) = \{F^{\pm}, \bar{F}^{\pm}\} \ni (u(1) \oplus sl(2; \iota_1))$ but with the subalgebra $sl(2; \iota_1) = \{H, E^{\pm}\}$ instead of $sl(2)$. Under contraction $j_1 = \iota_1$ we have the semidirect sum $sl(2; \iota_1; \nu|1) = \{E^{\pm}, F^{+}, \bar{F}^{-}\} \ni \{H, Z, F^{-}, \bar{F}^{+}\}$ of the subsuperalgebras, each of them generating both even and odd generators.

5. Casimir operators

The study of Casimir operators plays a great role in the representation theory of simple Lie algebras since their eigenvalues characterize representations. In the case of Lie superalgebras their eigenvalues completely characterize a typical representation while they are identically vanishing on an atypical representation. An element C of the universal enveloping superalgebra $U(A)$ commuting with all elements of $U(A)$ is called a Casimir operator of superalgebra A. The algebra of the Casimir operators of A is the Z_2-centre of $U(A)$.

The Casimir operators of the basic Lie superalgebras can be constructed as follows [21–23]. Let $A = sl(m|n)$ with $m \neq n$ or $osp(m|n)$ be a basic Lie superalgebra. Let $\{E_{IJ}\}$ be a matrix basis of the generators of A where $I, J = 1, \ldots, m+n$ with $\deg I = 0$ for $I = 1, \ldots, m$ and $\deg I = 1$ for $I = m+1, \ldots, m+n$. Then defining $(\bar{E})_{IK} = (-1)^{\deg K} E_{IK}$, a standard sequence of Casimir operators is given by

$$C_p = \mathrm{str}(\bar{E}^p) = \sum_{I=1}^{m+n} (-1)^{\deg I} (\bar{E}^p)_{II} = \sum_{I, I_1, \ldots, I_{p-1}=1}^{m+n} E_{II_1} (-1)^{\deg I_1} \ldots E_{I_k I_{k+1}} (-1)^{\deg I_{k+1}} \ldots E_{I_{p-1} I}. \tag{19}$$

In the case of $sl(m|n)$ with $m \neq n$ one finds, for example, $C_1 = 0$ and

$$C_2 = \sum_{i,j=1}^{m} E_{ij} E_{ji} - \sum_{\bar{k},\bar{l}=m+1}^{m+n} E_{\bar{k}\bar{l}} E_{\bar{l}\bar{k}} + \sum_{i=1}^{m} \sum_{\bar{k}=m+1}^{m+n} (E_{\bar{k}i} E_{i\bar{k}} - E_{i\bar{k}} E_{\bar{k}i}) - \frac{m-n}{mn} Y^2. \tag{20}$$

The diagonal elements of matrix $\bar{E}$ are taken in the form $(\bar{E})_{ii} = E_{ii} + \frac{1}{m} Y$, $(\bar{E})_{\bar{k}\bar{k}} = -E_{\bar{k}\bar{k}} + \frac{1}{n} Y$ and two conditions on the generators: $\sum_{i=1}^{m} E_{ii} = 0$, $\sum_{\bar{k}=m+1}^{m+n} E_{\bar{k}\bar{k}} = 0$ are taken into consideration. In the case of $osp(m|n)$ one finds $C_1 = 0$ and

$$C_2 = \sum_{i,j=1}^{m} E_{ij} E_{ji} - \sum_{\bar{k},\bar{l}=m+1}^{m+n} E_{\bar{k}\bar{l}} E_{\bar{l}\bar{k}} + \sum_{i=1}^{m} \sum_{\bar{k}=m+1}^{m+n} (E_{\bar{k}i} E_{i\bar{k}} - E_{i\bar{k}} E_{\bar{k}i}). \tag{21}$$

One has to stress that unlike the algebraic case, the centre of $U(A)$ for the classical Lie superalgebras is, in general, not finitely generated. For only Lie superalgebra $osp(1|2n)$ the centre of its universal enveloping superalgebra is generated by n Casimir operators of degree $2, 4, \ldots, 2n$.

To obtain Casimir operators of superalgebra $sl(m; j|n; \epsilon)$ we shall proceed in the standard manner. First, we get the matrix $\bar{E}(j; \epsilon)$. For this we put in matrix $\bar{E}$ the new generators of $sl(m; j|n; \epsilon)$ instead of the old ones of $sl(m; n)$ according to (15) and denote the obtained matrix as $\bar{E}(\rightarrow)$. In general its elements are undefined for nilpotent values of parameters j, ϵ, ν. So it is necessary to multiply $\bar{E}(\rightarrow)$ on the minimal multiplier which eliminates all undefined expressions in matrix elements, namely, $\nu(1, m)[1, n]$. Finally we have

$$\bar{E}(j; \epsilon) = \nu(1, m)[1, n] \bar{E}(\rightarrow) \tag{22}$$

with matrix elements ($k \neq p, \bar{k} \neq \bar{p}$)

$$(\bar{E}(j;\epsilon))_{kk} = \nu(1,m)[1,n]\left(E_{kk} + \frac{1}{m}Y\right) \qquad (\bar{E}(j;\epsilon))_{\bar{k}\bar{k}} = \nu(1,m)[1,n]\left(-E_{\bar{k}\bar{k}} + \frac{1}{n}Y\right)$$
$$(\bar{E}(j;\epsilon))_{kp} = \nu(1,k)(p,m)[1,n]E_{kp} \qquad (\bar{E}(j;\epsilon))_{\bar{k}\bar{p}} = \nu(1,m)[1,\hat{\bar{k}}][\hat{\bar{p}},n]E_{\bar{k}\bar{p}}$$
$$(\bar{E}(j;\epsilon))_{i\bar{k}} = -(i,m)[\hat{\bar{k}},n]E_{i\bar{k}} \qquad (\bar{E}(j;\epsilon))_{\bar{i}k} = (k,m)[\hat{\bar{i}},n]E_{\bar{i}k}. \tag{23}$$

The maximal multiplier $\nu(1,m)[1,n]$ has the diagonal elements and the minimal unit multiplier has the matrix elements $(\bar{E}(j;\epsilon))_{m,m+n} = E_{m,m+n}$, $(\bar{E}(j;\epsilon))_{m+n,m} = E_{m+n,m}$.

The sequence of Casimir operators of $sl(m;j|n;\epsilon)$ is given by

$$C_p(j;\epsilon) = \operatorname{str}\bar{E}^p(j;\epsilon) = \nu^p(1,m)^p[1,n]^p \operatorname{str}(\bar{E}(\rightarrow))^p. \tag{24}$$

Indeed, let $X^\star$ be an arbitrary generator of $sl(m|n)$. Under computing $[C_p, X^\star] = 0$ we get identical terms but with opposite signs (plus and minus) so their sum is equal to zero. Under transformation of this commutator to the corresponding commutator of $sl(m;j|n;\epsilon)$ identical terms are multiplied on identical multipliers, therefore, their sum remains equal to zero, i.e. $[C_p(j;\epsilon), X] = 0$.

Let us illustrate the above expressions with the simple example of the $sl(2;j_1|1)$ superalgebra. The generators are transformed as follows:

$$\begin{array}{llll} E_{11} = E_{11}^\star & Y = Y^\star & E_{12} = j_1 E_{12}^\star & E_{21} = j_1 E_{21}^\star \\ E_{13} = \nu E_{13}^\star & E_{31} = \nu E_{31}^\star & E_{23} = \nu j_1 E_{23}^\star & E_{32} = \nu j_1 E_{32}^\star \end{array} \tag{25}$$

and matrix $\bar{E}(j_1)$ according to (22)–(23) is given by

$$\bar{E}(j_1) = \nu j_1 \bar{E}(\rightarrow) = \nu j_1 \left(\begin{array}{cc|c} E_{11} + \frac{1}{2}Y & \frac{1}{j_1}E_{12} & -\frac{1}{\nu}E_{13} \\ \frac{1}{j_1}E_{21} & -E_{22} + \frac{1}{2}Y & -\frac{1}{\nu j_1}E_{23} \\ \hline \frac{1}{\nu}E_{31} & \frac{1}{\nu j_1}E_{32} & Y \end{array}\right)$$
$$= \left(\begin{array}{cc|c} \nu j_1\left(E_{11} + \frac{1}{2}Y\right) & \nu E_{12} & -j_1 E_{23} \\ \nu E_{21} & \nu j_1\left(-E_{22} + \frac{1}{2}Y\right) & -E_{23} \\ \hline j_1 E_{31} & E_{32} & \nu j_1 Y \end{array}\right). \tag{26}$$

The first-order Casimir operator disappears $C_1(j_1) = \operatorname{str}\bar{E}(j_1) = 0$. The second-order Casimir operator is as follows:

$$C_2(j_1) = \operatorname{str}(\bar{E}(j_1))^2 = \nu^2 j_1^2\left(2E_{11}^2 - \tfrac{1}{2}Y^2\right) + \nu^2\left(E_{12}E_{21} + E_{21}E_{12}\right)$$
$$+ j_1^2\left(E_{31}E_{13} + E_{13}E_{31}\right) + E_{32}E_{23} - E_{23}E_{32}. \tag{27}$$

In the case of superalgebras $osp(M|N)$ the multiplier in (23) equals $\nu(1,M)\left[1,\frac{N}{2}\right]$ and all formulae for matrix $\bar{E}(j;\epsilon)$ and matrix elements $(\bar{E}(j;\epsilon))_{kp}$ appear as for the $sl(m;j|n;\epsilon)$ with substitutions $m = M$ and $n = \frac{N}{2}$. Let us consider the $osp(1|2;\nu)$ superalgebra as an example. Its generators are transformed as

$$E_{12} = \nu E_{12}^\star \qquad E_{13} = \nu E_{13}^\star \qquad E_{23} = E_{23}^\star \qquad E_{32} = E_{32}^\star \qquad E_{22} = E_{22}^\star \tag{28}$$

and matrix $\bar{E}(\nu)$ is given by

$$\bar{E}(\nu) = -\nu\left(\begin{array}{c|cc} 0 & \frac{1}{\nu}E_{12} & \frac{1}{\nu}E_{13} \\ \hline \frac{1}{\nu}E_{13} & E_{22} & E_{23} \\ -\frac{1}{\nu}E_{12} & E_{32} & -E_{22} \end{array}\right) = -\left(\begin{array}{c|cc} 0 & E_{12} & E_{13} \\ \hline E_{13} & \nu E_{22} & \nu E_{23} \\ -E_{12} & \nu E_{32} & -\nu E_{22} \end{array}\right). \tag{29}$$

The first-order Casimir operator equals zero, $C_1(\nu) = \operatorname{str}\bar{E}(\nu) = 0$ and the second-order Casimir operator is represented as

$$C_2(\nu) = \nu^2 E_{22}^2 + (E_{12}E_{13} - E_{13}E_{12}) - \tfrac{1}{2}\nu^2\left(E_{32}E_{23} + E_{23}E_{32}\right). \tag{30}$$

6. Conclusion

Using classical CK Lie algebras of different types we have built basic CK superalgebras. Unlike the standard procedure [18] of zero-tending parameter contractions in this work are described with the help of nilpotent-valued parameters. Such an approach gives an opportunity to obtain the distribution of contraction parameters among superalgebra generators starting from quadratic form, and hence, to build CK superalgebras by means of pure algebraic tools without a limiting procedure. Contracted superalgebras are connected with transformations of superspaces with nilpotent Cartesian coordinates and represent a wide class of different semidirect sums for different possible contractions. Infinite sequences of Casimir elements of CK superalgebras have been obtained by suitable transformations of the standard expressions of the corresponding operators of the basic superalgebras. It is our hope that CK superalgebras will be relevant for the construction of supersymmetric physical models.

Acknowledgment

This work was supported by the Russian Foundation for Basic Research under project 01-01-96433.

References

[1] Golfand Yu A and Likhtman E P 1971 *JETP Lett.* **13** 452
[2] Volkov D V and Akulov V P 1972 *JETP Lett.* **16** 621
[3] Wess J and Zumino B 1974 *Nucl. Phys.* B **70** 139
[4] West P 1986 *Introduction to Supersymmetry and Supergravity* (Singapore: World Scientific)
[5] Keck B W 1975 *J. Phys. A: Math. Gen.* **8** 1819
[6] Hatsuda M and Sakaguchi M 2001 *Preprint* hep-th/0106114, KEK-TH-769 and DAMPT-2001-50
[7] Vasiliev M A 1990 *Phys. Lett.* B **243** 378
[8] Bars I 1996 *Preprint* hep-th/9608061
[9] Bellucci S, Ivanov E and Krivonos S 2001 *Nucl. Phys.* B (*Proc. Suppl.*) **102–103** 26
[10] Gromov N A 1990 *Contractions and Analytical Continuations of Classical Groups. Unified Approach* (Syktyvkar: Komi SC) (in Russian)
[11] Gromov N A and Man'ko V I 1990 *J. Math. Phys.* **31** 1054, 1060
[12] Rembielinski J and Tybor W 1984 *Acta Phys. Pol.* B **15** 611
[13] Hussin V, Negro J and del Olmo M A 1999 *J. Phys. A: Math. Gen.* **32** 5097
[14] Patra M K and Tripathy K C 1989 *Lett. Math. Phys* **17** 1
[15] de Montigny M and Patera J 1991 *J. Phys. A: Math. Gen.* **24** 525
[16] Moody R V and Patera J 1991 *J. Phys. A: Math. Gen.* **24** 2227
[17] Celeghini E, Giachetti R, Kulish P P, Sorace E and Tarlini M 1991 *J. Phys. A: Math. Gen.* **24** 5675
[18] Inönü E and Wigner E P 1953 *Proc. Natl Acad. Sci. USA* **39** 510
[19] Gromov N A, Kostyakov I V and Kuratov V V 2002 *Quantum Theory and Symmetries (Proc. II Int. Symp. (Krakow, Poland, 18–21 July 2001))* (River Edge, NJ: World Scientific) p 360 (*Preprint* hep-th/0110257)
[20] Gromov N A 1990 *Int. J. Theor. Phys.* **29** 607
[21] Frappat L, Sciarrino A and Sorba P 1996 Dictionary on Lie superalgebras *Preprint* hep-th/9607161, ENSLAPP-AL-600/96 and DSF-T-30/96
[22] Leites D and Sergeev A 2002 *Preprint* math.RT/0202180 v1
[23] Arnaudon D, Chryssomalakos C and Frappat L 1995 *Preprint* q-alg/9503021 v2, ENSLAPP-A-505/95